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Preface

This volume of the journal is a tribute to the mathematical contributions of Thomas H. Brylawski (1944–2007). We begin with a survey of Tom's life and his mathematics by two of his former Ph.D. students. Tom worked in matroid theory and made particularly important contributions to the study of matroid constructions, the Tutte polynomial, and matroid representation theory. All of these topics feature repeatedly throughout this volume.

Matroids were introduced by Whitney [13] in 1935 to capture the fundamental properties of dependence that are common to matrices and graphs. Tom began working in matroid theory barely thirty years later. In the more than forty years since then, many major advances have taken place in the subject. The excluded-minor characterizations of ternary [1,9] and quaternary matroids [2] have been proved; Seymour [10] described the structure of regular matroids and thereby found a polynomial-time algorithm for testing total unimodularity of a matrix; the Tutte polynomial has been shown to have important applications in even more areas than previously realized including knot theory [12] and statistical mechanics [11]; and the Graph Minors Project of Robertson and Seymour (see, for example, [7]) has been extended to binary matroids by Geelen et al. (see [3] for a survey of their work).

The last extension marks a particular milestone in the subject. One of the aims of Tom's advisor, Gian-Carlo Rota, was to put combinatorics onto a more equal footing with other, more established branches of mathematics. Combinatorics had often been characterized as a collection of *ad hoc* special tricks; Rota sought to develop a solid theoretical basis for the subject that matched other mathematical disciplines in its sophistication. His series of ten papers *On the Foundations of Combinatorial Theory* (surveyed in [6]) was an initial attempt to do this. For graph theory and matroid theory, the Graph Minors Project and its generalization to matroids representable over finite fields have done what Rota was aiming to achieve. They have established deep and solid foundational cores for these areas that anyone working in the areas needs to understand.

It is an exciting time to be working in matroid theory, matching the excitement of the 1960s when the theory was being extensively studied on both sides of the Atlantic: at MIT, Waterloo, Oxford, and Paris. Just as the Graph Minors Project has opened the way to massive new developments in graph theory, one can expect the ongoing work of Geelen, Gerards, and Whittle to have a corresponding effect on matroid theory. These exciting new developments build on earlier foundational work in the subject. Tom Brylawski was one of the leading figures in matroid theory to emerge from that burst of interest in the subject in the 1960s and the effect of his contributions continues to be felt.

Tom's interest in matroid constructions began with his work on series and parallel connection and on generalized parallel connection. The operations of 2-sum for matroids and 3-sum for binary matroids are easily derived from these. In the current volume, three papers are devoted to constructions for matroids, those of Bonin and Schmitt, of Hochstättler and Nickel, and of Servatius and Servatius. One of Tom's lasting contributions to the subject is his extensive survey of matroid constructions.

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The Tutte polynomial was another interest of Tom's from very early on. The study of this and related polynomials has grown dramatically. In the present volume, five papers are devoted to such polynomials and corresponding invariants, namely, those of Diao, Hetyei, and Hinson, of Ellis-Monaghan and Sarmiento, of Kung and Royle, of Lawrence, and of Traldi. The breadth of coverage in these papers exemplifies the widespread reach of the Tutte polynomial.

Matroid representation theory has always been a core area of study in the subject and one in which the problems are renowned for their difficulty. Tom's result with Dean Lucas that ternary matroids are uniquely $GF(3)$-representable is a key property of such matroids and, indeed, the presence of inequivalent representations of matroids over $GF(q)$ for larger values of q has been extensively studied in attempts to prove Rota's Conjecture [8] that the set of excluded minors for the class of $GF(q)$-representable matroids is finite for all q. The two longest papers in this volume give excluded-minor characterizations of two basic classes. Motivated by a question of Tom, the paper of Mayhew, Oporowski, Oxley, and Whittle treats the union of the classes of binary and ternary matroids, while Hall, Mayhew, and van Zwam consider the class of near-regular matroids, those that are representable over all fields other than possibly $GF(2)$.

The subject of extremal matroid theory was surveyed twenty years ago by Kung [5]. This area has seen significant recent growth with the widespread use of Ramsey-theoretic methods, and a highlight of this development is Geelen et al.'s proof [4] of Kung's Growth-Rate Conjecture. In the current volume, the papers of Geelen and of Chun and Oxley represent two aspects of work in this area. Another aspect of matroid theory, how a typical n-element matroid can be expected to behave, is the subject of the paper of Mayhew, Newman, Welsh, and Whittle. It contains numerous interesting conjectures.

The combinatorics group at MIT played a key role in the life of Tom Brylawski. It is therefore appropriate that this volume should include contributions from two of Tom's graduate-student contemporaries, Curtis Greene (with Cuttler and Skandera) and Richard Stanley.

Tom Brylawski's enthusiasm for mathematics was infectious. We hope that the papers in this tribute volume stimulate its readers to continue the exciting journey of mathematical discovery to which Tom helped introduce both of us.

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